Integrated Assessment of Energy Policies:

A Decomposition of Top-Down and Bottom-Up

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Outline of Presentation

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1. Motivation

Top-Down versus Bottom-Up

- Top-down models:
  - Analysis of the broader economy with policy-induced feedback
  - Sparse representation of technologies in energy production or conversion

- Bottom-up models:
  - Here: energy system models cast as optimization models
  - Discrete representation of technological options
  - Omission of macroeconomic feedbacks
  - Neglect of potentially important second-best aspects of markets

Hybrid Modeling: Top-Down and Bottom-Up

Hybrid modeling aims to combine the technological explicitness of bottom-up models with the economic nuance of top-down models:

- The “soft-link approach”: e.g., Hofman and Jorgenson (1976), Hogan and Weyant (1982), Drouet et al. (2005), and Schaefer and Jacoby (2006)).
- The “reduced form approach” – one model is supplemented with a short-hand representation of the other: e.g. Manne (1977), Manne and Richels (1994), Bahn, Kypreos, Büeler and H. J. Luethi (1999), Messner and Schrattenholzer (2000), Bosetti et al. (2006).
- The “integrated approach” based on complementarity: e.g. Böhringer (1998), Böhringer, Müller and Wickart (2003), Böhringer and Rutherford (2007).

Motivation and Contribution

- Motivation:
  - Integrated MCP approach provides flexibility and overall consistency but
  - Complexity and dimensionality limit its practical application.

- Contribution:
  - Decomposition method in which complementarity methods are used to solve the top-down economic equilibrium model and quadratic programming is applied to solve the underlying bottom-up energy (supply) model.
  - Complementarity and optimization “subproblems" can be specified and solved within modeling languages, eliminating the need for extensive programming.
  - Rapid convergence of our iterative (Jacobi) algorithm relies on the fable of the elephant and the rabbit: the energy sector should be a small fraction of GDP.

2. Integrated Model Formulation

Large Scale Applications: Many Technologies

Key features of the model:

- Top-down economic system (endogenous capital formation, potentially several sectors)
- Bottom-up energy system (competitive firms, engineering-based, technology-oriented, detailed)

Variables in the economic model:

p	a nonnegative n-vector of prices for all goods and factors in the economy
y	a nonnegative m-vector of activity levels for constant-returns-to-scale (CRTS) production sectors
M	an h-vector of income levels for all "consumers" (representing households and government entities)

Bottomup Linkages to Top Down

- e represents a non-negative n-vector of net energy system outputs (including, for example, electricity, oil, coal, and natural gas supplies to residential, industrial, and commercial customers), and
- x denotes a non-negative n-vector of energy system inputs (including labor, capital, and materials inputs).

Economic Equilibrium

Zero profit:

$$-\Pi_j(p) \geq 0 \quad \perp \quad y \geq 0$$

Market clearance:

$$\sum_j \nabla \Pi_j(p)\ y_j + \sum_k \omega_k + e \geq \sum_k d_k(p, M_k) + x$$

Income balance:

$$M_k = p^T \left[\ \omega_k + \theta_k(e - x)\ \right]$$

Competitive, Profit-Maximizing Energy Sector

e and x solve:

$$\max p^T(e - x)$$

subject to

$$Ax + Bz \geq Ce$$

$$e, x \geq 0, \quad \ell \leq z \leq u$$

in which $A, C \in R^{M \times n}$, and $B \in R^{M \times N}$ characterize technological constraints and $z \in R^N$ denotes decision variables of the energy system.

Attribution of Energy Rents

$$M_k = p^T \left[\, \omega_k + \theta_k(e - x) \,\right] = p^T \omega_k + \theta_k(\mu^T u + \lambda^T \ell)$$

Model Dimensions

m economic activities
n energy goods
M energy system LP constraints
N ancillary energy system LP decision variables ($z \in R^N$)

Equation Counts

Integrated MCP model: $m + 3n + h + M + 3N$
Economic model (without the energy system): $m + n + h$
LP energy model: M constraints and $N + 2n$ variables

Integrated (Simultaneous) Solution

MCP: KKT conditions of the energy system LP joint with (macro-) economic equilibrium conditions.

$$C^T \pi \geq p, \quad e \geq 0, \quad e^T (C^T \pi - p) = 0$$

$$p \geq A^T \pi, \quad x \geq 0, \quad x^T (p - A^T \pi) = 0$$

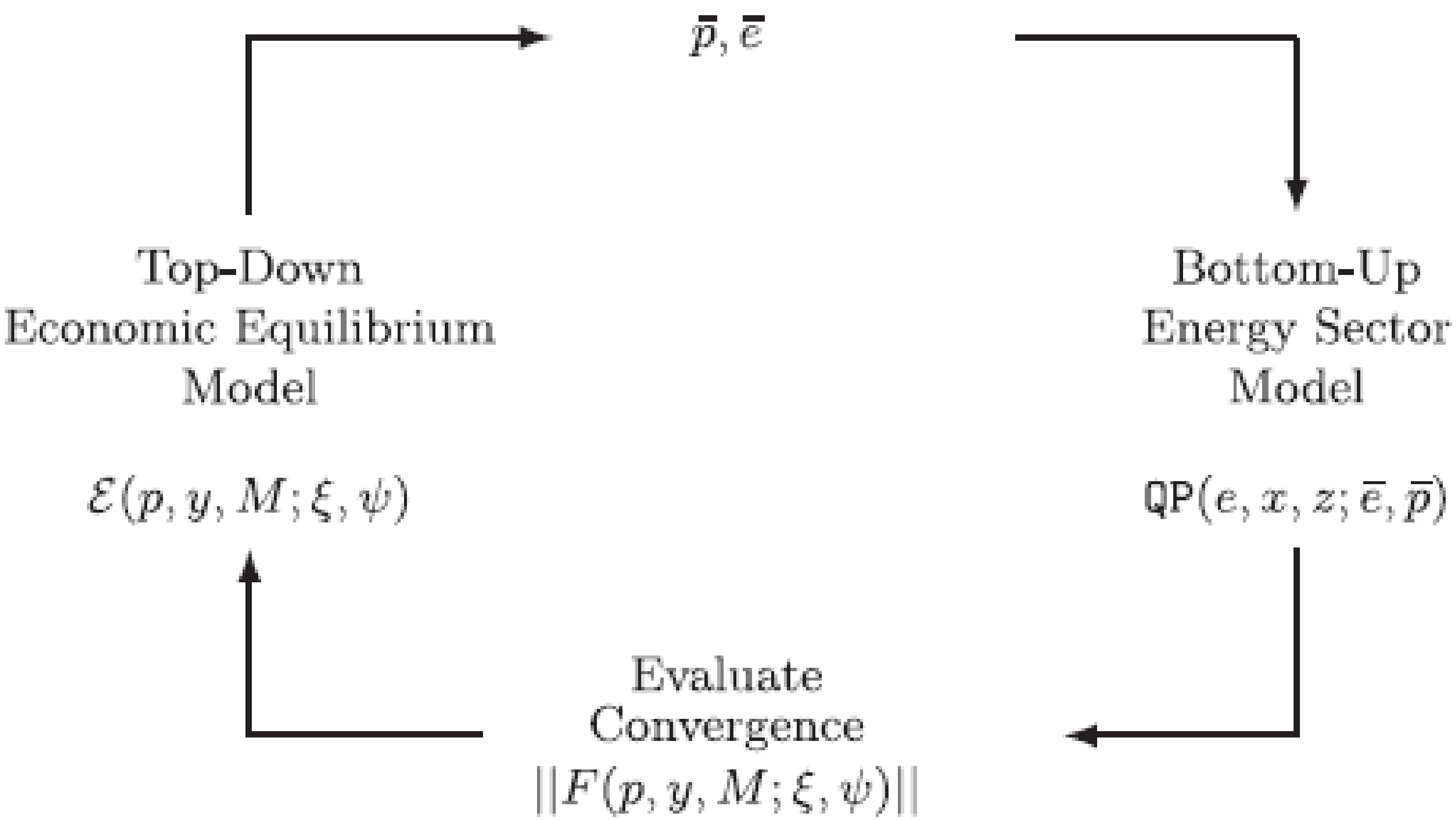
$$Ax + Bz \geq Ce, \quad \pi \geq 0, \quad \pi^T (Ax + Bz - Ce) = 0$$

$$\ell \leq z \leq u, \quad \lambda, \mu \geq 0, \quad \lambda (z - \ell) = 0, \quad \mu (u - z) = 0$$

$$\lambda + B^T \pi = \mu$$

3. Decomposition

Iterative Decomposition Algorithm



Constructing the Approximation

Demand for energy good i is represented through a Marshallian demand system:

$$e_i(p) = \bar{e}_i[1 - \epsilon_i(p_i/\bar{p}_i - 1)]$$

where ϵ_i is the elasticity of demand and e_i and p_i are observable reference quantities and prices for the demand function calibration.

Analytical Implementation

- Algorithm:
  - Computed equilibrium prices are p¯ (based on an initial guess for the energy sector response e¯, x¯, and q¯).
  - We then update the values of (e-x) and θ_k based on p¯ within the QP where we use a calibrated (diagonal) demand system:

$$e_i(p) = \bar{e}_i \left[1 - \epsilon_i(p_i/\bar{p}_i - 1)\right]$$

which has an inverse demand function:

$$p_i(e) = \bar{p}_i \left[1 + (1 - e/\bar{e}_i)/\epsilon_i\right]$$

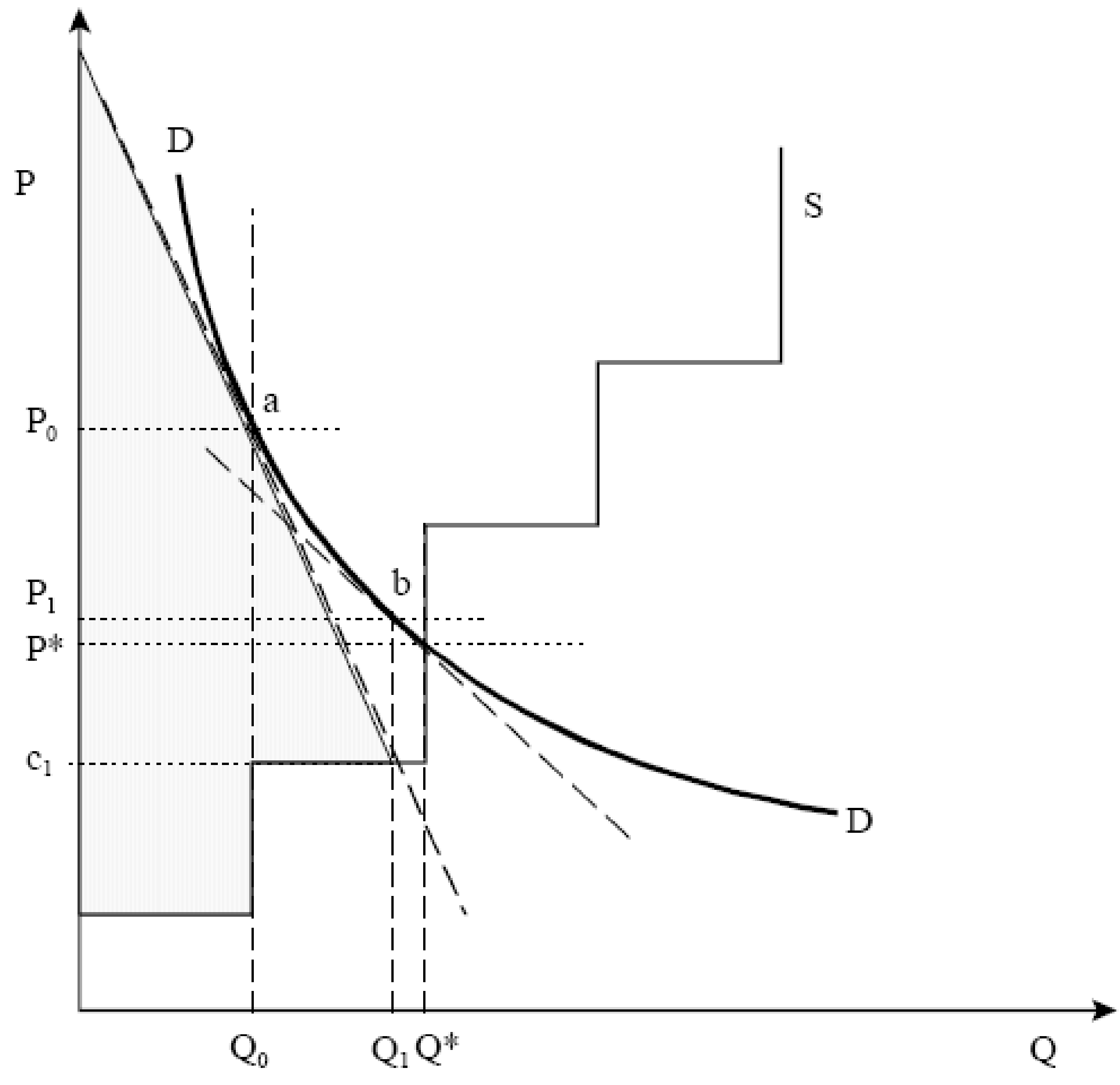
and an integrated market demand function:

$$\int p_i(e)de = \bar{p}_i e_i \left[1 - \frac{e_i - 2\bar{e}_i}{2\epsilon_i \bar{e}_i}\right]$$

  - Then solve the QP:

$$\max \bar{p}^T (e - x) - \frac{1}{2} \sum_i \frac{\bar{p}_i e_i}{\varepsilon_i \bar{e}_i} (e_i - 2\bar{e}_i)$$

Partial Equilibrium Illustration



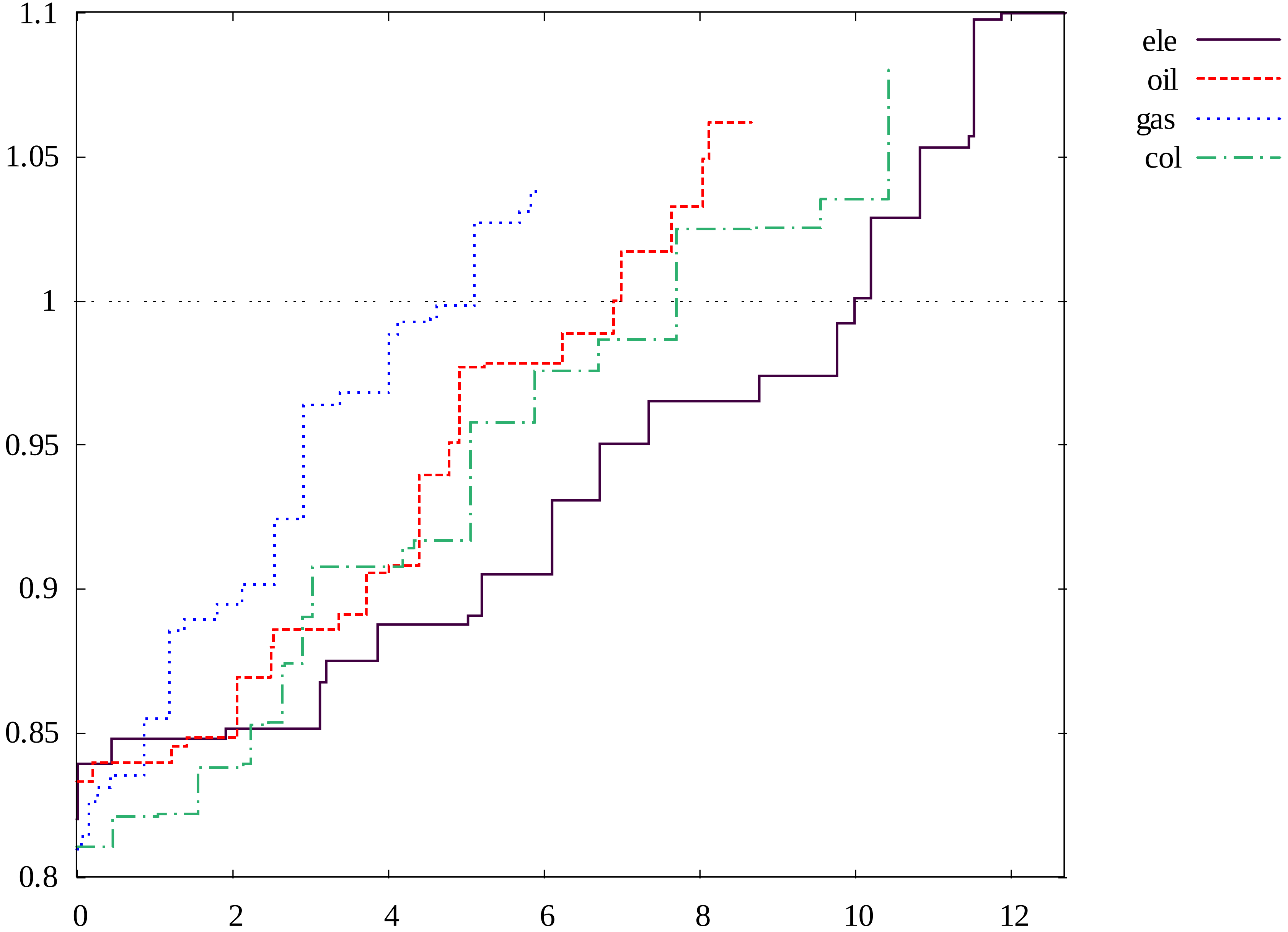
4. Illustration

Stylized Social Accounting Matrix

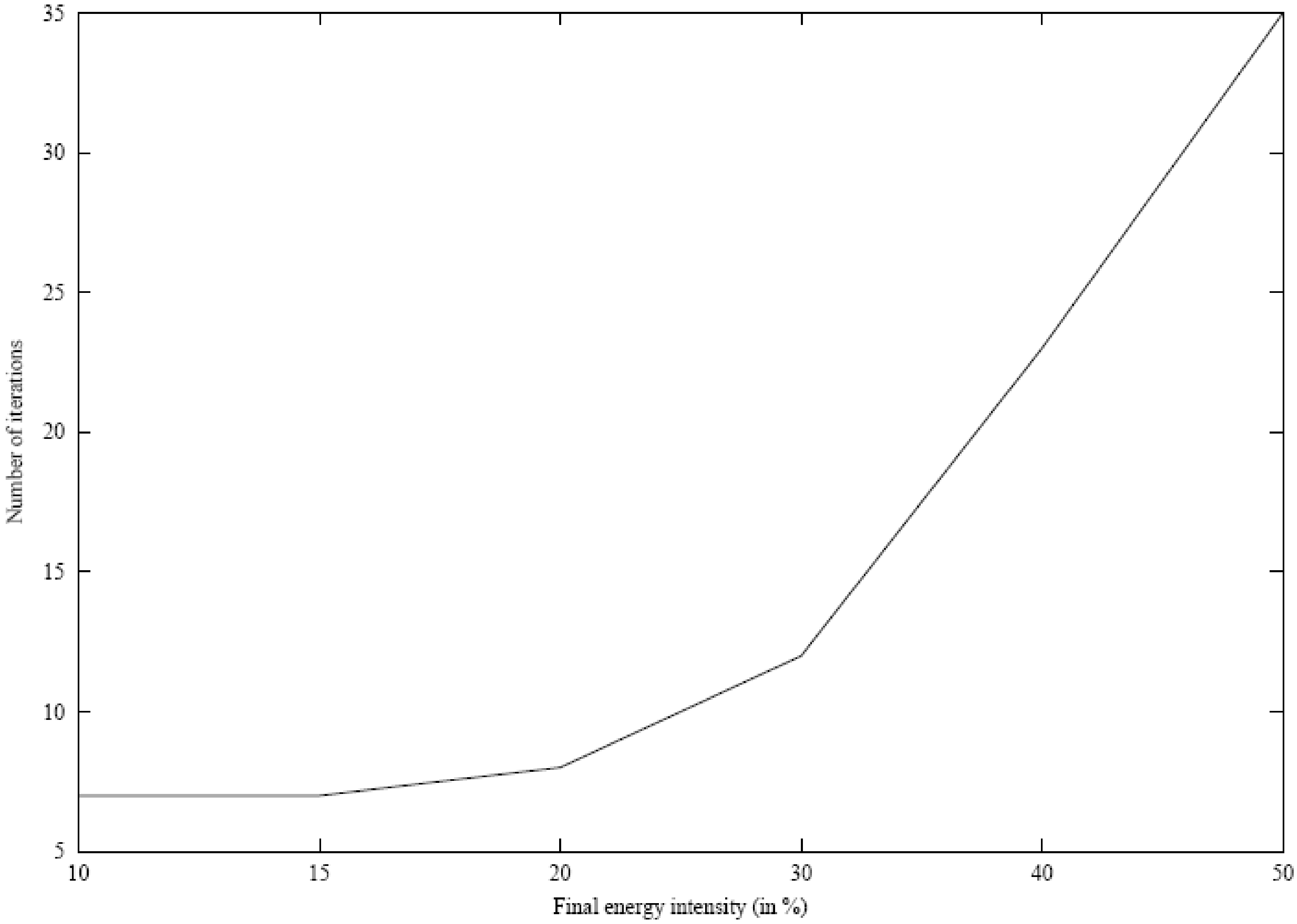
	x	y	e	fd
x	15			-15
y		100	-24	-76
l	-5	-40		45
k	-5	-50		55
ele	-2	-5	10	-3
oil	-1	-2	7	-4
gas	-1	-1	3	-1
col	-1	-2	4	-1

Key	
x:	energy intensive production
y:	macro production
e:	energy production
fd:	final demand
ele:	electricity
oil:	oil
gas:	gas
col:	coal

Step-wise Energy Supply Structure



Sensitivity to Energy Value Share



5. Conclusions

Summary

- Decomposition provides a means of interfacing models which work at different levels of aggregation and ceteris paribus assumptions:
  - MCP representation of the broader economy accounting for important real-world imperfections (inefficiencies)
  - Optimization approach to energy system with lots of details (bounds) at the technological level

- Quadratic programming provides an effective scheme for integrating bottom-up linear programming sub-models into a general equilibrium framework:
  - Sequential quadratic programming allow for a robust approximation of general equilibrium demand system in a partial equilibrium subproblem (provided the energy value shares are small).
  - Optimization and complementarity "subproblems" can be specified and solved within modeling languages, eliminating the need for extensive programming.

Addendum: Calibrated linear demand function

Demand function based on observed price/quantity pair ($\bar{p}$, $\bar{q}$) and (negative) elasticity ε

$$q(p) = \bar{q}(1 + \varepsilon(\frac{p}{\bar{p}} - 1))$$

Conventional linear demand function:

$$q(p) = a - bp \quad (*)$$

Observation benchmark market demand:

$$\bar{q} = a - b\bar{p} \qquad (1)$$

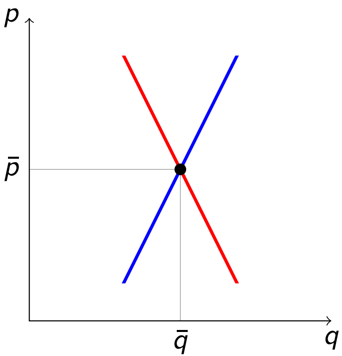
Price elasticity of demand at benchmark point:

$$\varepsilon = \frac{dq}{dp}\frac{\bar{p}}{\bar{q}} = -b\frac{\bar{p}}{\bar{q}} \qquad (2)$$

(1) → (1'): $a = \bar{q} + b\bar{p}$; (1) → (2'): $-b = \varepsilon\frac{\bar{q}}{\bar{p}}$

(1'), (2') in (*): $q = \bar{q} - \varepsilon\frac{\bar{q}}{\bar{p}}(\bar{p} - p) = \bar{q}(1 + \varepsilon(\frac{p}{\bar{p}} - 1))$

Calibrated Supply and Demand Functions



Calibrated Supply and Demand Functions

Given the following *data*:

$\bar{q}$ Reference quantity supplied (and demanded)

$\bar{p}$ Reference demand price

$\bar{\mu}$ Reference supply price

ϵ Magnitude of the price elasticity of demand

η Magnitude of the price elasticity of supply

We can write the demand and supply functions as:

$$d(p) = \bar{d}\left(1 - \epsilon\left(\frac{p}{\bar{p}} - 1\right)\right)$$

and

$$s(\mu) = \bar{s}\left(1 + \eta\left(\frac{\mu}{\bar{\mu}} - 1\right)\right)$$